Math 2050, HW 4 (due: 8 Nov)

Q1. If $\sum_{i=1}^{\infty} a_n$ with $a_n > 0$ is convergent

    (a) show that $\sum_{n=1}^{\infty} a_n^3$ is convergent;

    (b) Is $\sum_{n=1}^{\infty} a_n^{1/3}$ necessarily convergent? Prove it or provides a counter-example;

    (c) If $b_n = \frac{1}{n} \sum_{i=1}^{n} a_i$, show that $\sum_{n=1}^{\infty} b_n$ is divergent.

Q2. Show that

    (a) $\lim\limits_{x \to 1} \dfrac{x^3 - 2}{3 + x} = -\dfrac{1}{4}$;

    (b) $\lim_{x \to 0+} x^{1/4} \cos(e^{1/x}) = 0$.

Q3. Show that $\lim\limits_{x \to 1} \exp\left\{ \dfrac{1}{\sqrt{x} - 1} \right\}$ does not exist.

Q4. Let $: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. If f has a limit L at $x = 0$.

    (a) Show that $L = 0$;

    (b) Show that f has a limit at every $c \in \mathbb{R}$.